C2

1.) ALGEBRA & FUNCTIONS

⇒ Simplify using factorising

Eg. $$\frac{2x^2+11x+12}{(x+3)(x+4)} = \frac{(2x+3)\cancel{(x+4)}}{(x+3)\cancel{(x+4)}} = \frac{2x+3}{x+3}$$

⇒ Division of Polynomials

Eg. $(x^3-3x-2) \div (x-2)$

$$\begin{array}{r|l} & x^2+2x+1 \\ x-2 & x^3+0x^2-3x-2 \\ - & x^3-2x^2 \\ & \quad 2x^2-3x \\ - & \quad 2x^2-4x \\ & \qquad x-2 \\ & \qquad x-2 \\ & \qquad\quad 0 \end{array}$$

Remember to have all powers

i. Divide $\frac{x^3}{x} = x^2$

ii $x^2 \times (x-2)$

iii. Divide $\frac{2x^2}{x} = 2x$

iv. $2x \times (x-2)$

v. Divide $\frac{x}{x} = 1$

vi. $1 \times (x-2)$

* x^2+2x+1 = the Quotient
* $x-2$ ⇒ remainder $= 0$ ∴ $(x-2)$ is a factor ⇒ This is the factor theorem.

If $f(x)$, & $f(a)=0$
polynomial ∴ $(x-a)$ is a factor.

⇒ Synthetic method for LINEAR ONLY

Eg. $(2x^4-9x^3+13x^2-17x+15) \div (x-3)$

$x-3=0$
∴ $x=3$ →

3	2	−9	13	−17	15
+	0	6	−9	12	−15
	2	−3	4	−5	0

always have a 0 here

2×3 $-9+6$

shows it is a factor

i. make a grid using the coefficients of x

ii. add the factor on the opposite side

iii. add a '0' under the first coefficient & add.

↳ 2 −3 4 −5 0

↳ $\underbrace{(2x^3-3x^2+4x-5)}_{\text{Quotient}}(x-3) = (2x^4-9x^3+13x^2-17x+15)$

Eg. 2) $x^3 - 3x - 2 \div x - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -3 & -2 \\ \hline & 0 & 2 & 4 & 2 \\ & 1 & 2 & 1 & 0 \end{array}$$

$\therefore (x^2 + 2x + 1)(x - 2)$

Eg. 3.) $2x^3 - 5x^2 - 16x + 10 \div x + 4$

Not the best example

$$\begin{array}{r|rrrr} -4 & 2 & -5 & -16 & 10 \\ \hline & 0 & -8 & -52 & -272 \\ & 2 & -13 & -68 & -262 \end{array}$$ → remainder

$2x^2 - 13x - 68$ & remainder -262

* This is a very easy and quick way. Learn it because it will save you lots of time

* To check if $(x - a)$ is a factor of $f(x)$, check if $f(a) = 0$

Eg. Show that $(x-2)$ is a factor of $f(x) = x^3 + x^2 - 5x - 2$

→ If it is a factor $x - 2 = 0$

$\therefore x = 2$

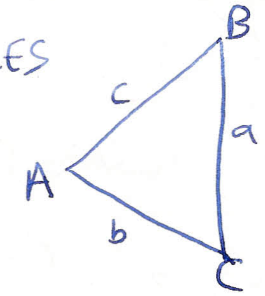
$$\begin{aligned} f(2) &= 2^3 + 2^2 - 5(2) - 2 \\ &= 8 + 4 - 10 - 2 \\ &= 0 \end{aligned}$$

$\therefore (x-2)$ if a factor of $f(x) = x^3 + x^2 - 5x - 2$

2.) TRIGONOMETRY & CIRCLES

→ SINE RULE

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$



→ COSINE RULE

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A} \Rightarrow \cos A = \frac{b^2+c^2-a^2}{2bc}$$

→ Area of any Triangle = $\boxed{\frac{1}{2}ab\sin C}$

→ TRIGONOMETRY IDENTITIES

* $\boxed{\tan x \equiv \frac{\sin x}{\cos x}}$

* $\boxed{\sin^2 x + \cos^2 x \equiv 1}$

$$\boxed{\begin{array}{l} \Rightarrow \sin^2 x = 1 - \cos^2 x \\ \Rightarrow \cos^2 x = 1 - \sin^2 x \end{array}}$$

Eg. $4\cos^2 x + \cos x + 1 = 2\sin^2 x$

$4\cos^2 x + \cos x + 1 = 2(1-\cos^2 x)$

$4\cos^2 x + \cos x + 1 = 2 - 2\cos^2 x$

$6\cos^2 x + \cos x - 1 = 0$

Use $\cos x = k$

$\therefore 6k^2 + k - 1 = 0$

$(3k-1)(2k+1) = 0$

$\therefore k = \frac{1}{3} ; \frac{-1}{2}$

$\therefore \cos x = \frac{1}{3}$, $\cos x = \frac{-1}{2}$

$\therefore x = 70.53°$ & $289.47°$, $\therefore x = 120°$ & $240°$

→ Equation of a Circle

- Centre (a, b) ; radius r

$$\boxed{(x-a)^2 + (y-b)^2 = r^2}$$ ← always rearrange equations into this form

- Tangent gradient m_1 × Normal/Perpendicular gradient m_2 = −1
- $y - y_1 = m(x - x_1)$

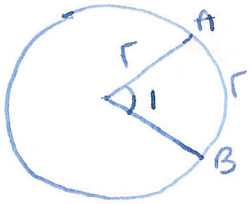
→ Midpoint = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

→ $\boxed{\text{distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$

} Remember C1 Geometry

→ Radian Measure

- Unit to measure angles
- Radian = angle subtended at the centre of a circle by an arc whose length is equal to radius of circle.



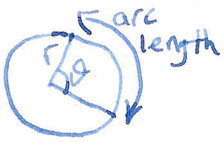
Units = 1 radian ; 1 rad or 1^c

$r = 1^c$

$2r = 2^c$

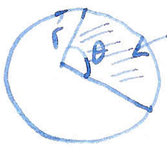
* $2\pi r = 360°$ $2\pi r = 2\pi^c$ → ∴ $\boxed{360° = 2\pi^c}$

→ Arc length :- Propotional to angle subtended at circle.



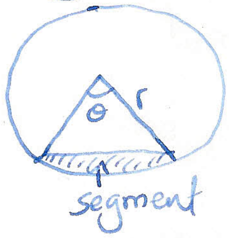
∴ $\boxed{\text{arc length} = \theta r}$ * θ = in radians, r = radius

→ Area of Sector = $\boxed{\frac{1}{2}r^2\theta}$ * θ = In radians, r = radius



$$1° = \frac{\pi}{180}$$

→ Segment = region bound by a chord & an arc



$$\text{Area of Segment} = \text{Area of Sector} - \text{Area of Triangle}$$

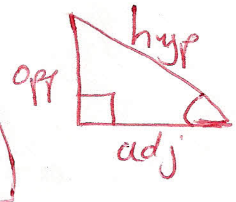
$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

$$= \boxed{\frac{1}{2}r^2(\theta - \sin\theta)}$$

* θ = In radians
r = radius

→ Special Values

* $\boxed{\sin = \frac{opp}{hyp}}$; $\boxed{\cos = \frac{adj}{hyp}}$; $\boxed{\tan = \frac{opp}{adj}}$



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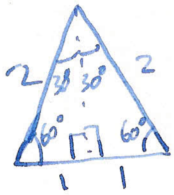
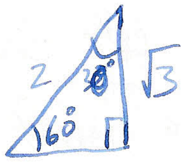
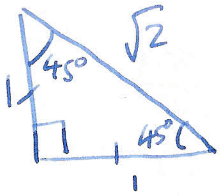






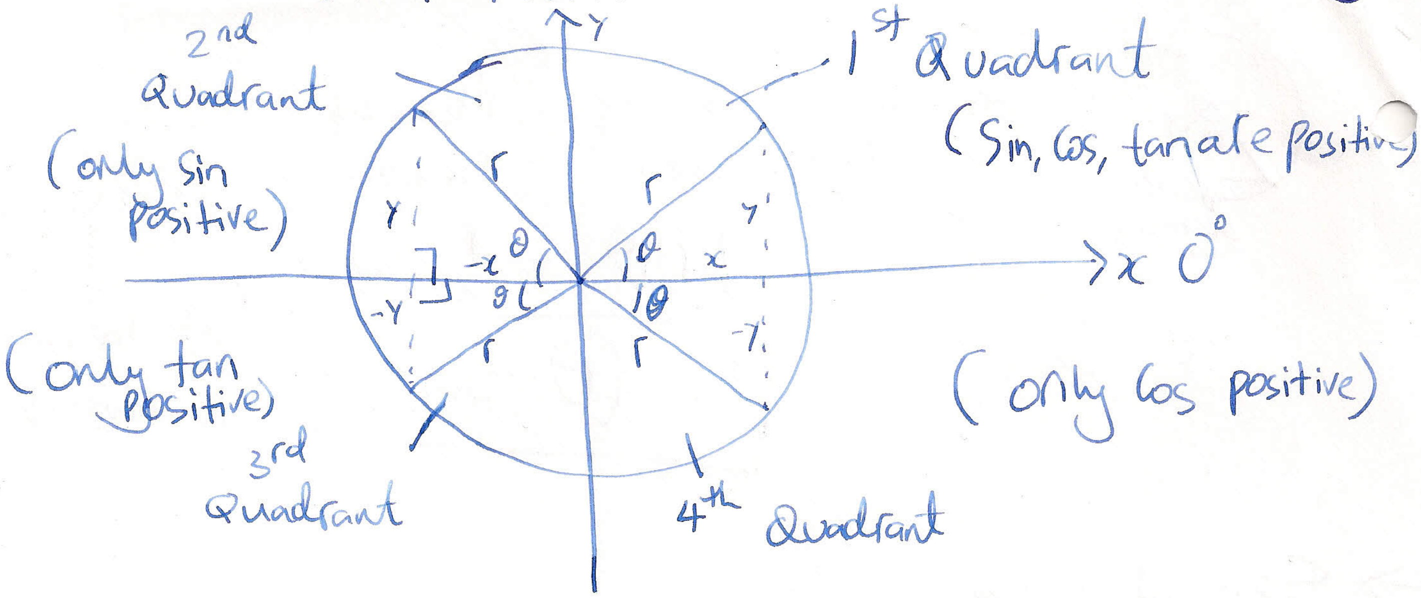
→ $\sin 60° = \frac{\sqrt{3}}{2}$; → $\sin 30° = \frac{1}{2}$; → $\sin 45° = \frac{\sqrt{2}}{2}$

→ $\cos 60° = \frac{1}{2}$; → $\cos 30° = \frac{\sqrt{3}}{2}$; → $\cos 45° = \frac{\sqrt{2}}{2}$

→ $\tan 60° = \sqrt{3}$; → $\tan 30° = \frac{\sqrt{3}}{3}$; → $\tan 45° = 1$

⇒ $\sin 60° = \sin\left(\frac{\pi}{3}\right)^c$; ⇒ $\sin 30° = \sin\left(\frac{\pi}{6}\right)^c$; ⇒ $\sin 45° = \sin\left(\frac{\pi}{4}\right)^c$

→ TRIGONOMETRY RATIOS



2nd Quadrant

- $\sin\theta = \frac{y}{r}$ (+)
- $\tan\theta = \frac{y}{-x}$ (−)
- $\cos\theta = \frac{-x}{r}$ (−)

1st Quadrant

- $\sin\theta = \frac{y}{r}$ (+)
- $\tan\theta = \frac{y}{x}$ (+)
- $\cos\theta = \frac{x}{r}$ (+)

3rd Quadrant

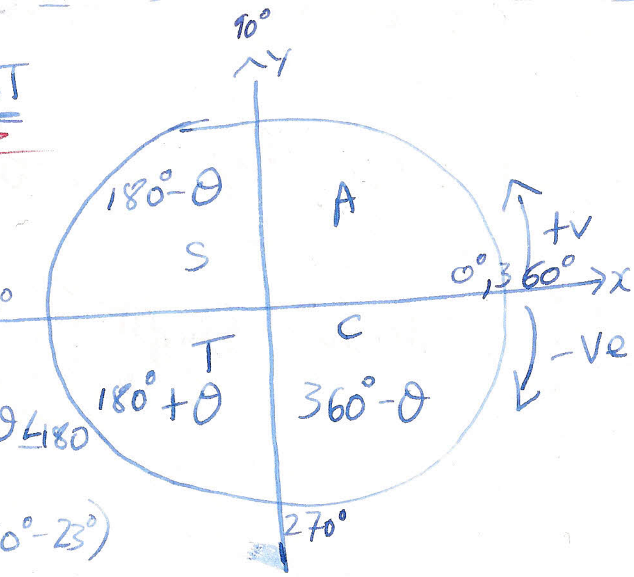
- $\sin\theta = \frac{-y}{r}$ (−)
- $\cos\theta = \frac{-x}{r}$ (−)
- $\tan\theta = \frac{-y}{-x}$ (+)

4th Quadrant

- $\sin\theta = \frac{-y}{r}$ (−)
- $\cos\theta = \frac{x}{r}$ (+)
- $\tan\theta = \frac{-y}{x}$ (−)

→ CAST

All
Soccer Player
Take
Cash



All
Silver
Tea
Cups

Eg. $\cos 2\theta = 0.921$

$0 < \theta < 180$

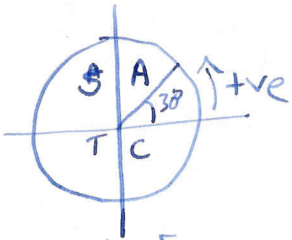
① $2\theta = 23°$
$\theta = 11.5°$

② $2\theta = 337°$ → $(360° - 23°)$
$\theta = 168.5°$

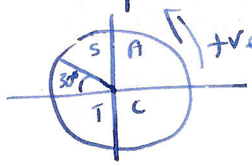
→ Eg. Find all values of x between $-360° < x < 360$

$\sin x = \frac{1}{2}$

* 1^{st} value ⇒ $\sin^{-1}\frac{1}{2} = 30°$ →

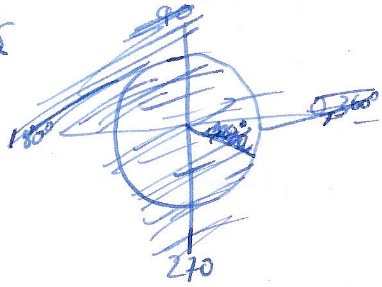


* 2^{nd} Value ⇒ $180° - 30° = 150°$ →

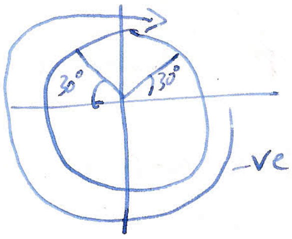


* 3^{rd} Value ⇒ For negative angles, take the first 2 value of x & minus $360°$

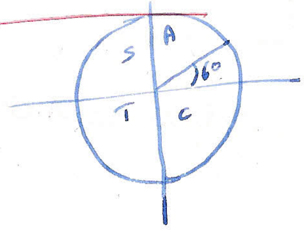
⇒ $30° - 360° = -330°$



* 4^{th} Value ⇒ $150° - 360° = -210°$



→ Eg. 2) Cosine $x = \frac{1}{2}$ $\quad -360° < x < 360°$



* 1^{st} Value ⇒ $\cos^{-1}\frac{1}{2} = 60°$

* 2^{nd} Value ⇒ $360° - 60° = 300°$

* 3^{rd} Value ⇒ $60° - 360° = -300°$

* 4^{th} value ⇒ $300° - 360° = -60°$

→ Eg. 3) $\tan x = -1$ $\quad -360° < x < 360°$

* 1st Value = $\tan^{-1} 1 = 45°$

* 2^{nd} Value = $180° + 45° = 225°$

* 3^{rd} Value = $45° - 360° = -315°$

* 4^{th} Value = $225° - 360° = -135°$

} For $\tan x = 1$

** One way is find the values of $\tan x = 1$ & ~~change~~ change the signs.

∴ $x = -45°, -225°, 315°, 135°$ → For $\tan x = -1$

3.) LOGS & EXPONENTIALS

$\boxed{\log_a b = c \Rightarrow a^c = b}$

a = base

$\boxed{\log_a a = 1 ; (a^1 = a)}$

$\boxed{\log_a 1 = 0 ; (a^0 = 1)}$

$\boxed{\log_a b^c = c \log_a b}$

$\boxed{\log_a X + \log_a Y = \log_a (XY)}$

$\boxed{\log_a X - \log_a Y = \log_a \left(\frac{X}{Y}\right)}$

* $\log_a \frac{1}{x} = -\log_a x$ * $\boxed{\frac{1}{x} = x^{-1}}$

→ Change Of Base

$$\boxed{\log_a x = \frac{\log_b x}{\log_b a}}$$

* Calculators Use base 10 ; $\log_{10} x = \log x$
  * normally 10 is not written.

Eg. Solve $2^{4x} = 3$ to 3dp.

* Write 'Log' on both sides
  - $\Rightarrow \log 2^{4x} = \log 3$
  - $\Rightarrow 4x \log 2 = \log 3$
* make x Subject of the formula & Solve

$$x = \frac{(\log 3)}{(4 \log 2)}$$

* remember to use brackets in your Calculator

$\therefore x = 0.396$ (to 3.d.p.)

4.) Sequence & Series

$$\boxed{{}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}}$$

* ! → Factorial.

Eg. $3! = 3 \times 2 \times 1$

$$\boxed{n! = n \times (n-1) \times (n-2) \times (n-3) \dots 3 \times 2 \times 1}$$

Eg. $${}^3C_2 = \frac{3!}{2!(3-2)!} = \frac{6}{2 \times 1} = 3$$

* $$\boxed{(1+x)^n = \binom{n}{0}1^n + \binom{n}{1}1^{n-1}x^1 + \binom{n}{2}1^{n-2}x^2 \dots + \binom{n}{r}1^{n-r}x^r}$$

$$= 1 + \frac{n}{1}x + \frac{n(n-1)}{1 \times 2}x^2 + \frac{n(n-1)(n-2)}{1 \times 2 \times 3}x^3 \dots$$

$$= 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \dots$$

* $$\boxed{(a+b)^n = a^n + \binom{n}{r}a^{n-r}b^r \dots + b^n}$$

* $$\boxed{(a+bx)^n = a^n + \binom{n}{r}a^{n-r}(bx)^r \dots + (bx)^n}$$

→ Geometric Progression

$$\boxed{U_n = ar^{n-1}}$$

a = 1st term
r = Common ratio

→ Sum of 1st n terms

$$S_n = a + ar + ar^2 + ar^3 + \dots ar^{n-1}$$

$(\times r)$ $rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$

$(-)$ $S_n - rS_n = a - ar^n$

$$S_n(1-r) = a(1-r^n)$$

$$\therefore \boxed{S_n = \frac{a(1-r^n)}{1-r}} \text{ or } \boxed{S_n = \frac{a(r^n-1)}{r-1}}$$

→ Convergent Series

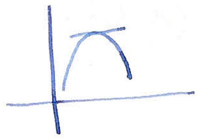
$$\boxed{S_\infty = \frac{a}{1-r}}$$

S_∞ = Sum to infinity

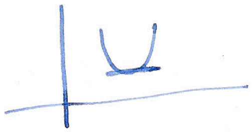
5) DIFFERENTIATION

- Stationary Point ⇒ gradient = 0 $\boxed{\frac{dy}{dx} = 0}$

→ $\boxed{\frac{d^2y}{dx^2} < 0}$ ⇒ maximum



→ $\boxed{\frac{d^2y}{dx^2} > 0}$ ⇒ minimum

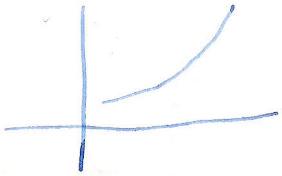


*Infexion Point

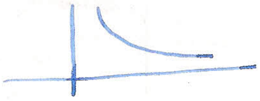
- $\frac{d^2y}{dx^2} = 0$
- $\frac{d^3y}{dx^3} \neq 0$

* → Functions

* Increasing – positive gradient



* Decreasing – Negative gradient



→ Curve sketching

* Find where the curve Crossed y & x axis
* $\boxed{dy/dx}$ Find gradient info. Check if it is increasing or decreasing
* $\boxed{d^2y/dx^2}$ Find minimum/maximum/Inflexion point
* Sketch the Curve

6.) INTEGRATION

→ Definite Integral

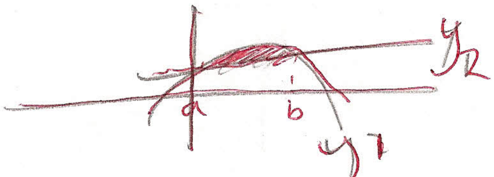
$$\int_a^b f'(x)\,dx = \Big[f(x)\Big]_a^b = f(b) - f(a)$$

Eg. $\int_1^3 (x^2+2)\,dx = \left[\frac{x^3}{3} + 2x\right]_1^3 = \left(\frac{3^3}{3} + 6\right) - \left[\frac{1^3}{3} + 2\right] = \frac{38}{3}$

* $\frac{1}{\infty} = \frac{1}{\infty^2} = \frac{1}{\infty^3} = 0$

make sure the area is between this region

→ Area between a line & Curve = $\boxed{\int_a^b (y_1 - y_2)\,dx}$



→ Area under a Curve = $\boxed{\int_a^b y\,dx}$

→ Trapezium Rule = $\boxed{\int_a^b y\,dx \approx \frac{1}{2}h\left[y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n\right]}$

width of each strip → $h = \boxed{\frac{b-a}{n}}$

* $\boxed{\text{5 y value (ordinates)} = \text{4 Strips}}$